Roll No. ....

CBD–5425-W

B. A./B. Sc./B. Sc. B. Ed.

(Fourth Semester) (End Semester)

EXAMINATION, 2022

MATHEMATICS

MTS–CC–411

(Algebra)

Time : Three Hours] [*Maximum Marks : 60*

Note : Attempt the questions from all three Sections as directed.

Section—A

(Objective Type Questions)

Note : Choose the correct answer. Each question carries 1 mark. 10×1=10

1. (i) The order of the symmetric group S_4 is :

(a) 6

(b) 12

(c) 18

(d) 24

(ii) In the following algebraic structure, which is not a semi-group ?

(a) (I, +)

(b) (I, .)

(c) (N, +)

(d) None of the above

(iii) The order of ω in the group $\left(\{1, \omega, \omega^2\}, .\right)$ is :

(a) 1

(b) 2

(c) 3

(d) 4

(iv) The number of finite subgroup in (Z, +) is :

(a) 1

(b) 4

(c) 11

(d) 0

(v) How many generators of the cyclic group G of order 10 ?

(a) 5

(b) 4

(c) 2

(d) 10

(vi) If H and K are finite subgroup of a group G, then o (HK) is :

(a) $\frac{o(H)}{o(K)}$

(b) $\frac{o(H).o(K)}{o(H \cap K)}$

(c) $\frac{o(H \cap K)}{o(H).o(K)}$

(d) None of the above

(vii) Every quotient group of a cyclic group is cyclic.

(a) Yes

(b) No

(c) Indefinite

(d) None of the above

(viii) Which of the following rings is not an integral domain ?

(a) (I, +, .)

(b) (Q, +, .)

(c) (R, +, .)

(d) None of the above

(ix) I (n) is a field, when :

(a) $n = 7$

(b) $n = 8$

(c) $n = 14$

(d) $n = 16$

(x) How many symmetries of a square are there ?

(a) 4

(b) 6

(c) 7

(d) 8

Section—B

(Short Answer Type Questions)

Note : Attempt any *four* questions. Each question carries 5 marks. $4 \times 5 = 20$

2. Prove that any group G of order 3 is cyclic.

3. Prove that every subgroup of a cyclic group is cyclic.

4. Prove that the intersection of any two normal subgroups of a group is a normal subgroup. https://www.dhsgsu.com

5. If the system (R, +, .) be a ring R, then show that :

   (i) $a.0 = 0.a = 0, \forall a \in R$

   (ii) $a.(-b) = (-a).b = -(a.b), \forall a, b \in R$

6. With picture and words, describe each symmetry in D_3 (the set of symmetries of an equilateral triangle).

7. Solve the equation :

$$235x \equiv 54 \pmod 7$$

Section—C

(Short Answer Type Questions)

Note : Attempt any *three* questions. Each question carries 10 marks. $3 \times 10 = 30$

8. Show that the four matrices :

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

   form a multiplicative group. Is this abelian ?

9. Show that union of two subgroups is a subgroup if and only if, one is contained in the other.

10. Prove that the order of each subgroup of a finite group is a divisor of the order of the group.

11. If $f(x)$ and $g(x)$ are two non-zero polynomials of R [x], then show that :

    (i) $\deg [f(x) + g(x)] \le \max [\deg f(x), \deg g(x)]$

(ii) $\deg [f(x) \cdot g(x)] \leq \deg f(x) + \deg g(x)$

where R $[x]$ represent the set of polynomials over a ring R.

12. Prove that the set S_n of all permutations on n symbols is a finite non-abelian group of order $n!$ with respect the composite of mappings as the operations.