Roll No. .... **C**

CBE-830-T

B.A./B.Sc./B.Sc. B.Ed. Fifth Semester (End Semester) Examination Dec., 2018

MATHEMATICS

Paper - MTS-EC-513

(Linear Algebra)

Time : Three Hours] [Maximum Marks : 60

Note :– All questions are compulsory.

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Section-A

(Objective Type Questions) 10×1=10

1. Which one of the following is not a vector space over the field IR of real numbers :

   (a) Set of all 2 × 3 matrices with entries as real numbers

   (b) Set of all real sequences which are bounded

   (c) set of all polynomials $P(x)$ with integer coefficients

   (d) Set of all continuous functions over [0, 1]

2. Which one of the following is correct :

   (a) $U = \{(x, y, z) \in \mathbb{R}^3 : x + y + z = 0\}$ is a subspace of $\mathbb{R}^3$

   (b) $V = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1\}$ is a subspace of $\mathbb{R}^3$

   (c) $W = \{(x, y) \in \mathbb{R}^2 : y = x^2\}$ is a sub space of $\mathbb{R}^2$

   (d) $S = \{(x, y, z) \in \mathbb{R}^3 : x = z + 2\}$ is a sub space of $\mathbb{R}^3$

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5. If $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation such that T (1, 1) = (2, 5) and T (1, 0) = (1, 6) then T (2, 3) is equal to :

(a) (10, 5)

(b) (5, 10)

(c) (11, 5)

(d) (5, 11)

6. If $T : \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation defined by : T (x, y, z) = (x − y, 2z) then :

(a) Nullity of $T = 1$ and rank of $T = 2$

(b) Nullity of $T = 0$ and rank of $T = 3$

(c) Rank of $T = 1$ and Nullity of $T = 2$

(d) Nullity of $T = 3$ and rank of $T = 0$

7. For which one of the following value of 'c' the set of vectors {(1, 0, 2), (2, −1, 3), (1, 3, c^2)} is linearly independent :

(a) $c = \sqrt{6}$

(b) $c = -\sqrt{6}$

(c) $c = \sqrt{2}$

(d) None of these

8. Let V be a vector space overfield F and $S \subseteq V$ then span $S = L(S)$ is the :

(a) largest sub space containing V

(b) largest sub space contained in S

(c) smallest sub space containing S

(d) smallest sub space contained in S

9. Choose the statement which is not correct :

(a) $\mathbb{R}^3$ is not isomorphic to $P_3(R)$

(b) $\mathbb{R}^4$ is isomorphic to $P_3(\mathbb{R})$

(c) $M_{2\times 2}$ is not isomorphic to $P_3(\mathbb{R})$

(d) $M_{2\times 2}$ is not isomorphic to $P_4(\mathbb{R})$

10. Choose the statement which is true :

(a) Eigen values must be non-zero scalars

(b) any two eigen vectors, of a linear operator are linearly independent

(c) Sum of two eigen values of a linear operator T is also an eigen value of T

(d) Let T be a linear operator on an n-dimensional vector space V with ordered basis β and if $[T]_\beta = A$ then both T and A have same eigen values.

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Section 'B'

(Short Answer Type Questions) 4x5=20

Note : Attempt any four questions.

1. Prove that a non-empty subset W of a vector space V is a subspace of V if and only if $ax \in W, x + y \in W$ whenever $a \in F$ and $x, y \in W$.

2. Define basis of a vector space. Test whether the set $S = \{(1, 0, -1), (2, 5, 1) (0, -4, 3)\}$ forms a basis for $\mathbb{R}^3$.

3. If $T : P_2(R) \to P_3(R)$ defined by $T(f(x)) = x f(x) + f'(x)$ then proved that T is linear transformation, also find bases for $N(T)$ and $R(T)$ and verify dimension theorem.

4. Define dual space. If $\beta = \{(2, 1), (3, 1)\}$ bean ordered basis for $\mathbb{R}^2$ and suppose $\beta^* = \{f_1, f_2\}$ is the dual basis of $\mathbb{R}^{2^*}$ then find the definitions of f_1 and f_2.

5. Let V and W be finite-dimensional vector spaces with ordered bases β and γ respectively. Let $T : V \to W$ be linear. Then prove that T is invertible if and only if $[T]_\beta^\gamma$ is invertible and $[T^{-1}]_\gamma^\beta = \left([T]_\beta^\gamma\right)^{-1}$.

6. Let V and W be finite dimensional vector spaces (over the same field). Then prove that V is isomorphic to W if and only if dim (V) = dim (W).

Section 'C'

(Long Answer Type Questions) 3×10=30

Note : Attempt any three questions.

1. (a) Prove that $W_1 + W_2$ is a subspace of a vector space V that contains both W_1 and W_2, where W_1 and W_2 are given subspaces of V.

   (b) Define vector space. Test whether the set of all real valued functions $f(x)$ such that $f(x^2) = (f(x))^2$ is vector subspace of V-the space of all functions from $\mathbb{R}$ to $\mathbb{R}$.

2. (a) Prove that any two bases of a vector space has same number of elements.

   (b) Prove that $\{(1-2x-2x^2, -2+3x-x^2, 1-x+6x^2\}$ forms a basis for $P_2(\mathbb{R})$.

3. (a) Let V and W be finite dimensional vector spaces having ordered bases β and γ respectively and let $T: V \rightarrow W$ be linear then prove that for each $u \in V$ we have $[T(u)]_\gamma = [T]_\beta^\gamma [u]_\beta$.

   (b) Find the matrix representation of the linear transformation $T = \mathbb{R}^2 \rightarrow \mathbb{R}^3$ defined by $T(a_1, a_2) = (a_1 + 3a_2, 0, 2a_1 - 4a_2)$.

4. (a) Let T be a linear operator defined on $\mathbb{R}^2$ then find the eigen values of T, where, $T(a,b) = (-2a+3b, -10a+9b)$.

   (b) Prove that a finite dimensional vector space V is isomorphic to its double dual V^{**}.

5. (a) Let T be the linear operator on $\mathbb{R}^2$ defined by $T(x,y) = (3x-y, x+3y)$ and $\beta = \{(1,1),(1,-1)\}$ and $\beta' = \{(2,4),(3,1)\}$ be the ordered bases then find the co-ordinate matrix Q that changes β'-cordinates into β-cordinates. Also find the matrix representation of T with respect to β and β'.

   (b) Define invertible linear transf-