Roll No. ....

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CBC-601-T

B.A./B.Sc./B.Sc. B.Ed.

Third Semester (End Semester)

Examination Dec., 2018

MATHEMATICS

Paper - MTS-CC-311

(Real Analysis)

Time : Three Hours] [Maximum Marks : 60

Note :– All section are compulsory.

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Section-A

(Objective Type Questions) 10×1=10

Note : Choose the correct option.

1. If A is a non-empty subset of *R*, that is bounded below, than A has a :
   (a) lub is R
   (b) glb in *R*
   (c) upper bound in *R*
   (d) None of these

2. The set of all intervals with rational end points in :
   (a) Uncountable
   (b) Countable
   (c) countably infinite
   (d) None of these

3. R is complete if and only if the .... property is true :
   (a) greatest lower bound
   (b) lower bound
   (c) least upper bound
   (d) upper bound.

4. The sequence $(S_n) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + ... + \frac{1}{n} + ...$ is

(a) convergent

(b) Not convergent

(c) camely

(d) None of these

5. Let series for which $a_n = \frac{1}{n(n+1)}$, then $\sum a_n$ is :

(a) $\frac{1}{2}$

(b) 1

(c) 0

(d) None of these

6. The series $\sum\left(\frac{1}{n} + \frac{1}{2n}\right)$ is :

(a) convergent

(b) divergent

(c) oscillates

(d) bounded

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7. The series $1\frac{1}{2} - 1\frac{1}{4} + 1\frac{1}{8} - 1\frac{1}{16} + ...$ is an series whose terms form a

(a) monotomic increasing sequence

(b) monotomic decreasing sequence

(c) not monotomic decreasing sequence

(d) None of these

8. The series $\sum (-1)^n \left[\sqrt{n^2+1} - n\right]$ is :

(a) Absolutely convergent

(b) Conditional convergent

(c) convergent

(d) not convergent

9. Let $f_n(x)$ for all $x \in Q$ be sequence of real valued function. Then :

(a) The number of point sequence is zero

(b) The number of point sequence is finite

(c) The number of point sequence is countable infinite

(d) None of the above

10. Let $f_n(x) = \tan^{-1} nx, n \in [0,1]$, whose point-wise limit is :

$$f(x) = \begin{cases} 0 & x = 0 \\ \frac{\pi}{2} & n \in [0,1] \end{cases}$$

then

(a) $f(x)$ is continous

(b) $f(x)$ is not differentiable

(c) $f(x)$ is differentiable

(d) None of these

Section 'B'

(Short Answer Type Questions) 4×5=20

Note : Attempt any four questions.

11. Any infinite subset of a countable set is countable.

12. If x and y are real numbers and if $y > 0$, then there exists a natural number n such that $ny > x$.

13. Every bounded sequence of real numbers has convergent subsequence.

14. State and prove Bolzano Weierstrass theorem for sequences.

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15. If series $\sum a_n$ converges absolutely then the series $\sum a_n$ converges, but only conversly.

16. Show that the sequence $\{f_n\}$, where $f_n(x) = x^n$ is uniformly convergent in [0, k], where k is a number less than are and only point wise convergent in [0, 1].

Section 'C'

(Long Answer Type Questions) 3×10=30

Note : Attempt any three questions.

17. Define in details of least upper bound property of the real number. If A and B are the two non-empty subsets of R, let $C = \{a + \frac{b}{a} \in A, b \in B\}$. Then

(i) If each of A and B has a least upper bound, then C has the least upper bound and lub C = lub A + lub B.

(ii) If each of A and B has a greatest lower bound, then C has the greatest lower bound and glbC = glbA + glbB.

18. Define Cauchy sequences and A sequence (S_n) of real numbers is convergent if and only if is a cauchy sequence.

19. (a) The series $\sum a_n$ is convergent if and only if for every $\epsilon > 0$, there exist a $n_0 \in N$ such that $|a_n + a_{n+1} + \dots + a_m| < \epsilon$ for $m > n \geq n_0$.

(b) Find out whether the series $\sum \frac{\sin n\frac{\pi}{2}}{n}$ is in l^2.

20. Prove that the sequence $< x_n >$ where $x_n = \left[\sqrt{n+1} - \sqrt{n}\right] \forall n \in N$ is convergent.

21. Define uniform convergence and differentiability. If $< g_n >$ and $< f_n >$ are two sequence of function defined for $0 \leq x \leq 1$ by $g_n = \frac{nx}{1+n^3+x^2}$ and

$$f_n(x) = \frac{\{\log(1+n^3+x^2)\}}{2n^2}$$

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Prove that $< g_n >$ converges uniformly to zero on [0, 1] and hence obtain the uniform convergence of $< f_n >$.

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